Side-channel Attacks on Blinded Scalar Multiplications Revisited

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CARDIS

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Outline

Horizontal SCA attacks on ECC

Random-Order Elliptic Curves

Structured-Order Elliptic Curves

Concludion and Future Directions

Outline

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Elliptic Curves and Scalar Multiplication

- Let $\mathcal{E}$ an Elliptic Curve over $\mathbb{F}_p$.
- $Q = d[P]$
  with P, Q two points on $\mathcal{E}$ and d a scalar.
- E is the order of $\mathcal{E}$
  for all $P \in \mathbb{E}, E[P] = \mathcal{O}$.
- Scalar Multiplication is easy to compute (Double and Add algorithm).
- Scalar Multiplication Inverse is hard (ECDLP) $\Rightarrow$ Security of ECC.

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    else
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    end
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$$d = d + r \times E$$

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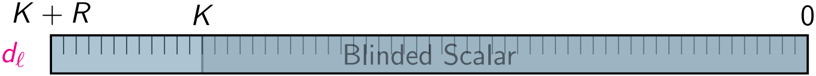
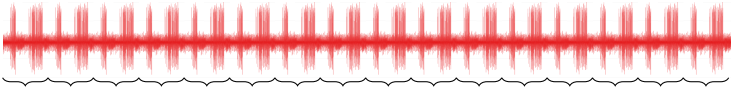


Horizontal SCA



T_ℓ

d_ℓ bits: 0 1 2 3 4 5 6 7 8 9 … … … … … …

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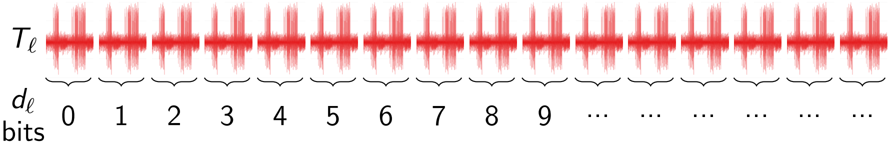

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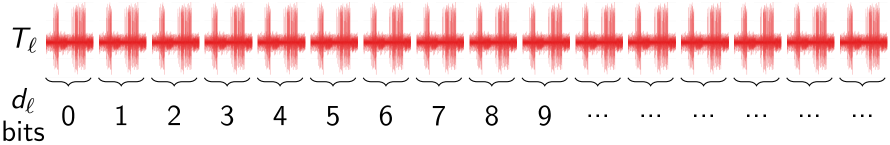
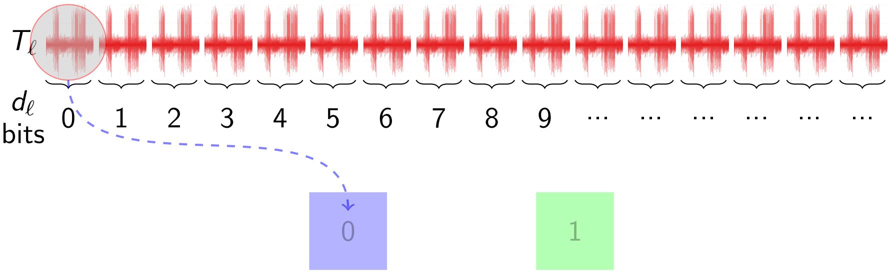


0

1

Horizontal SCA



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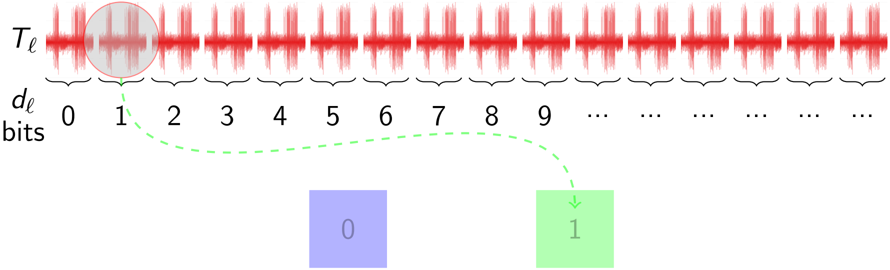

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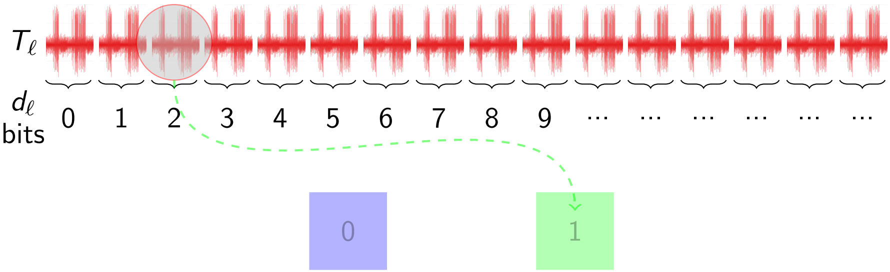

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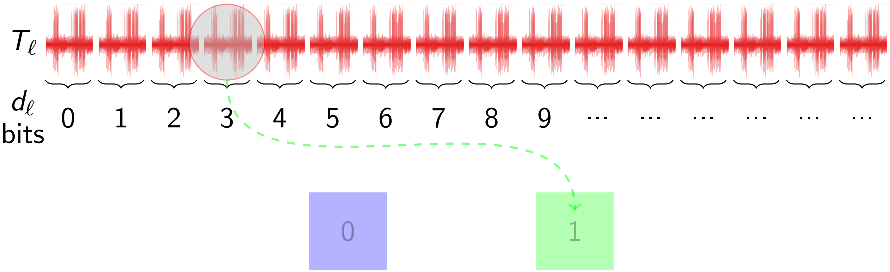

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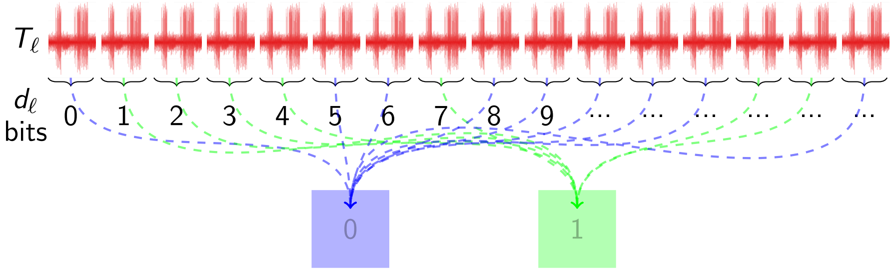

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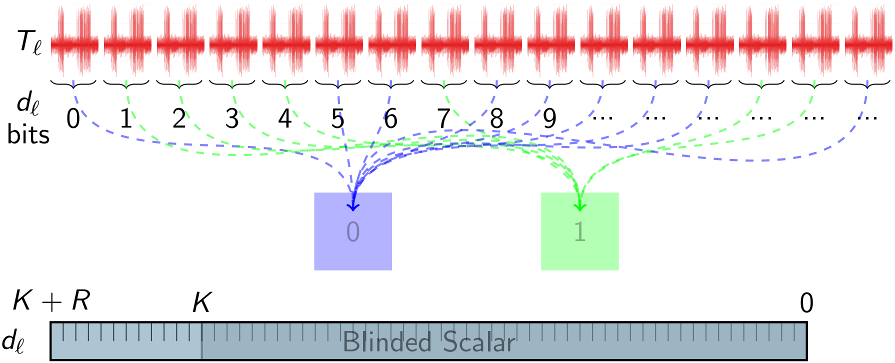

[Bauer *et al.* 2013]
[Weissbart *et al.* 2019]
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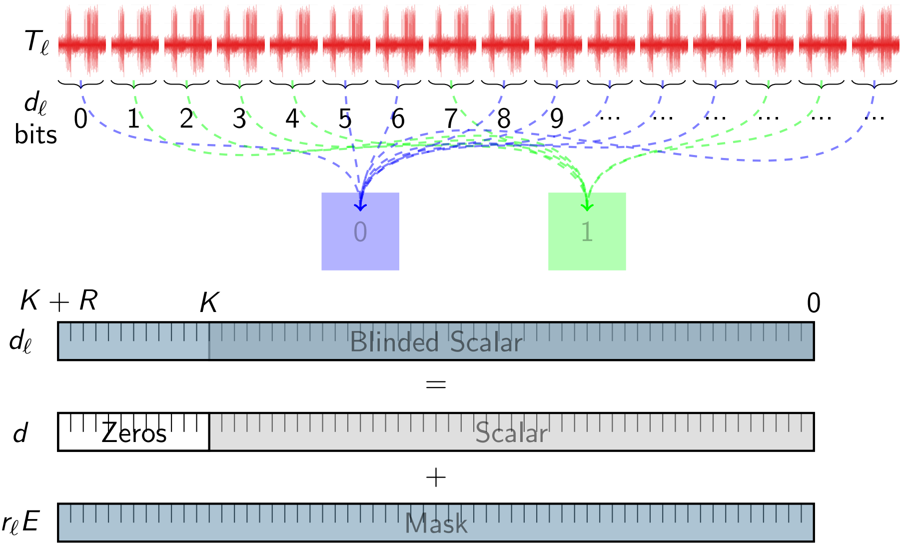

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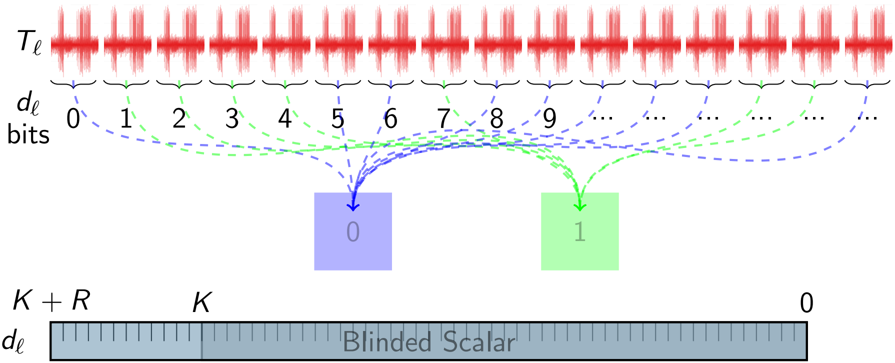

$$d = d_\ell \bmod E$$

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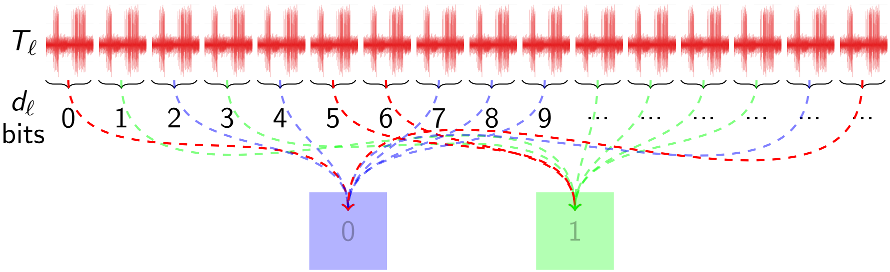

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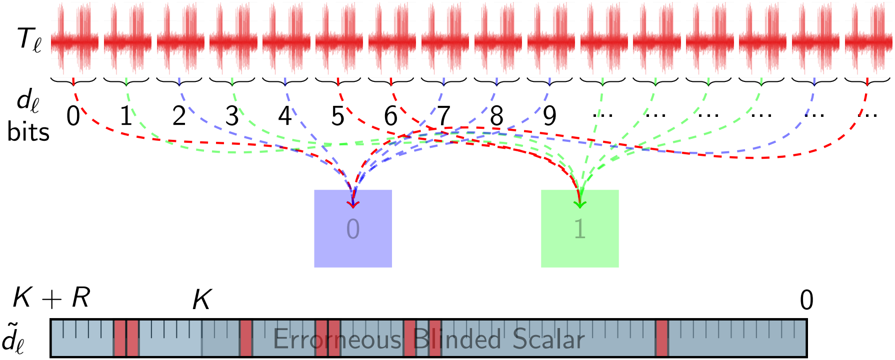

ϵ_b: bit-error probability

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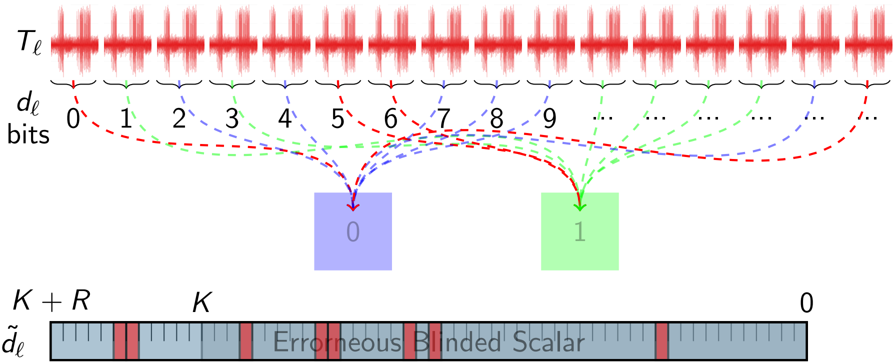

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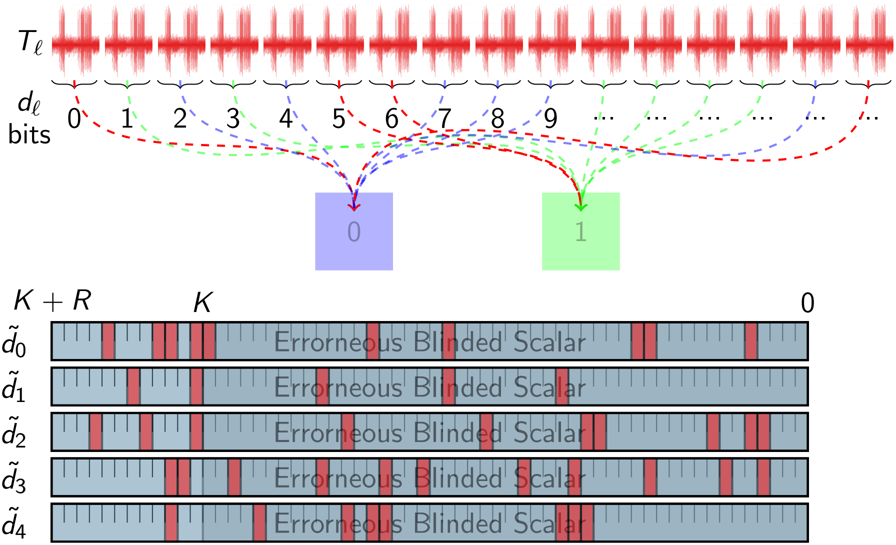

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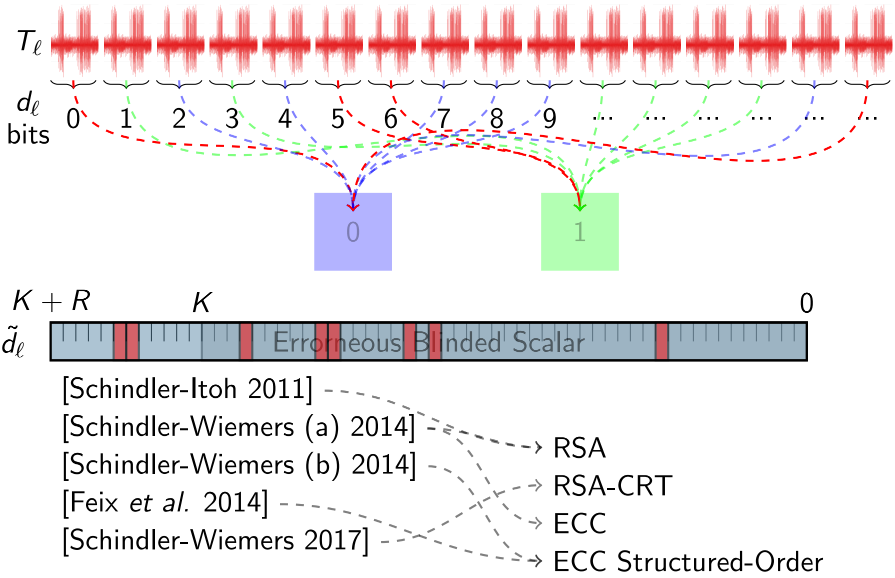

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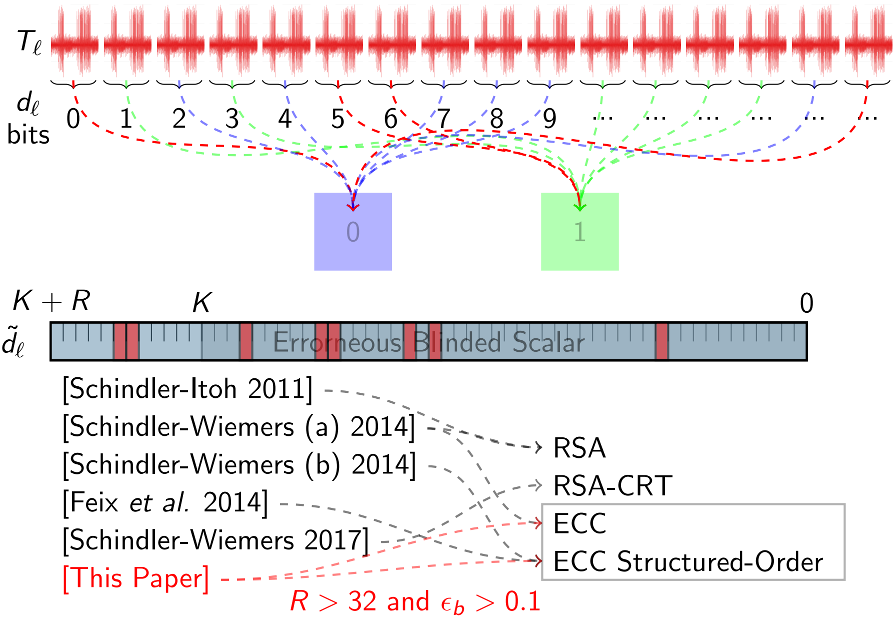

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NAF Distinguisher issue
$\rightarrow$ invalidates the contribution

currently working on a patch
eprint to be updated shortly

Outline

Structured-Order Elliptic Curves

- SEC2 curves (CERTICOM 2000)
- NIST curves (NIST FIPS 186-4)
- Curve25519 (Bernstein 2005)
- Brainpool curves (BSI RFC 5639)

$\Rightarrow$ **order E structured:** $E = 2^K \pm E_0$, where E_0 close to $2^{K/2}$

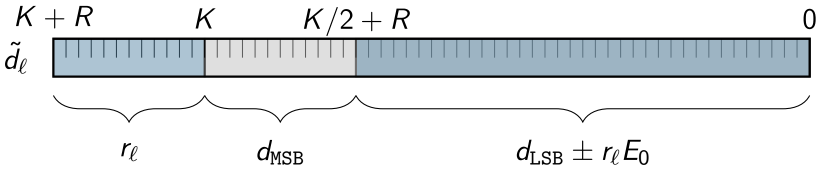
example - order of curve SEC2 secp256k1 (Bitcoin curve):

E = FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE BAAEDCE6 AF48A03B BFD25E8C D0364141

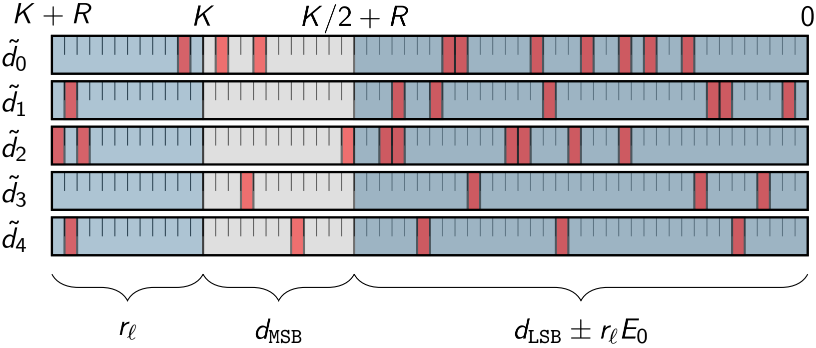
Structured-Order Effect on Scalar Randomization

- (d, E) of length K, r_ℓ of length R (with $R < K/2$)
- $d_\ell = d + r_\ell \times E$
- $E = 2^K \pm E_0$

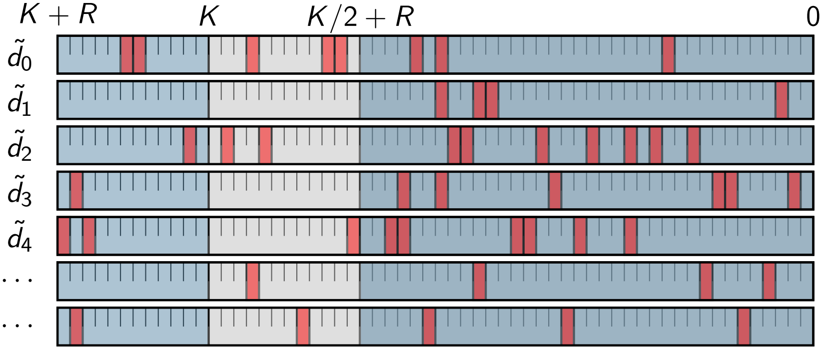
$$d_\ell = r_\ell \times 2^K + d \pm r_\ell \times E_0$$



Problem Definition



A divide and conquer algorithm [Schindler-Wiemers 2014]



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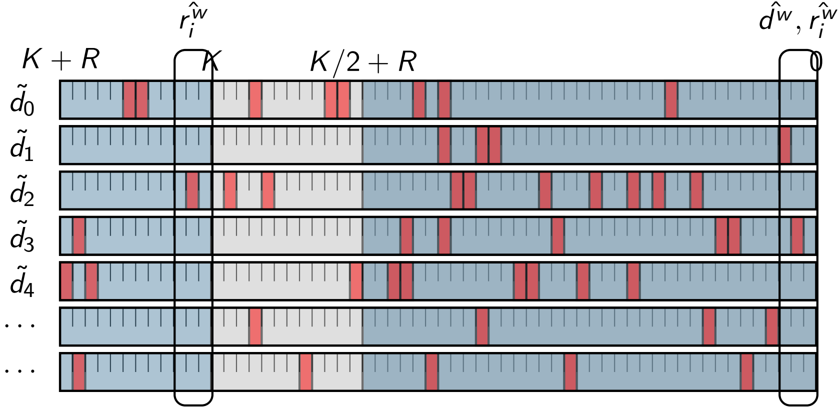

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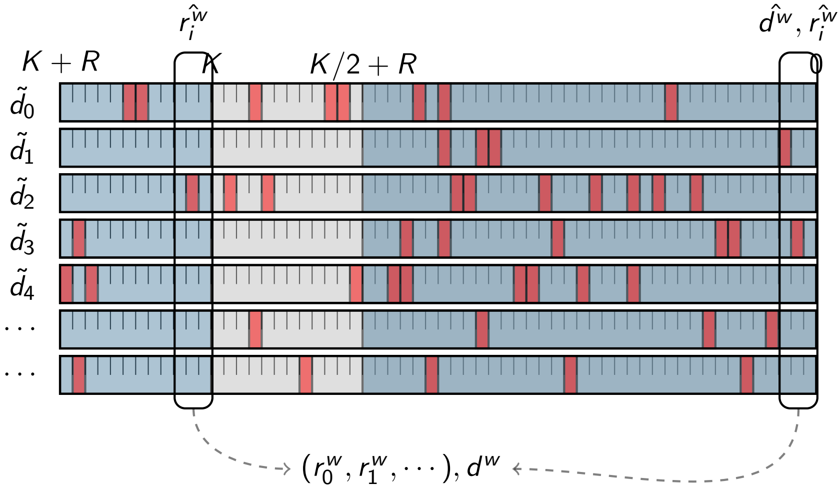

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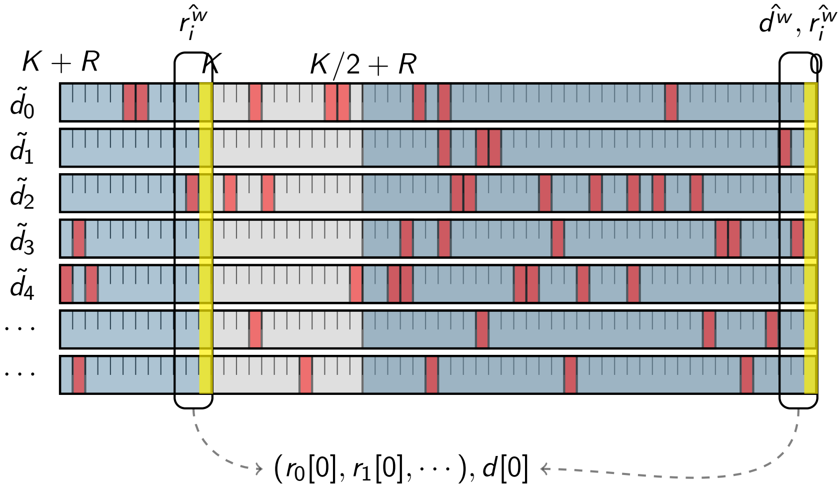

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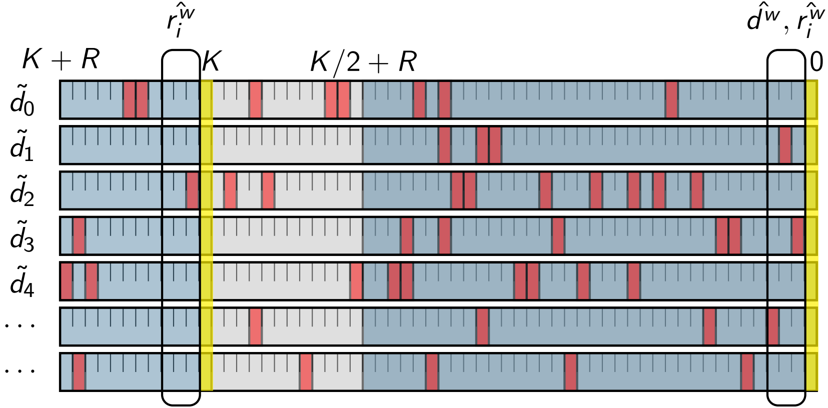

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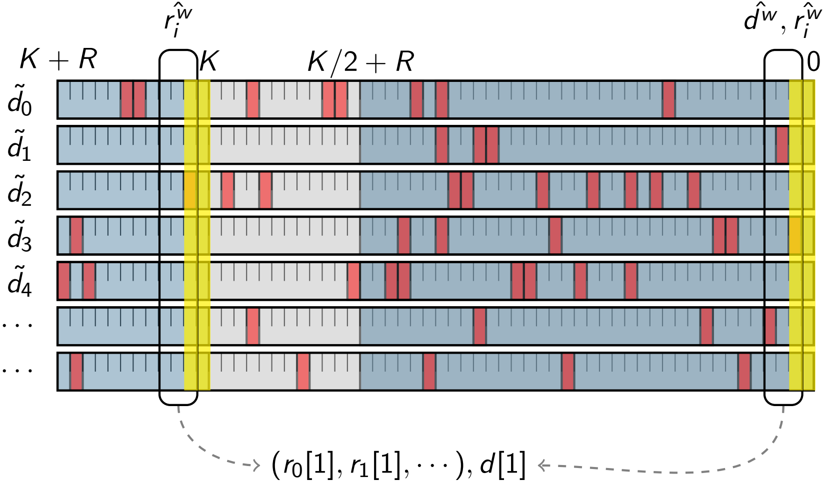

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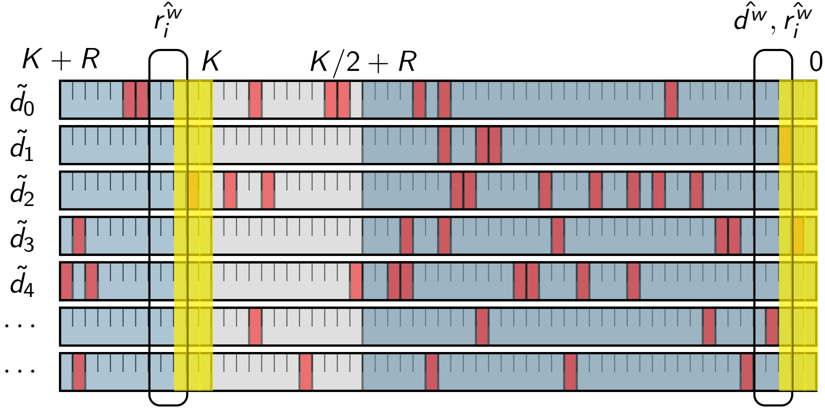

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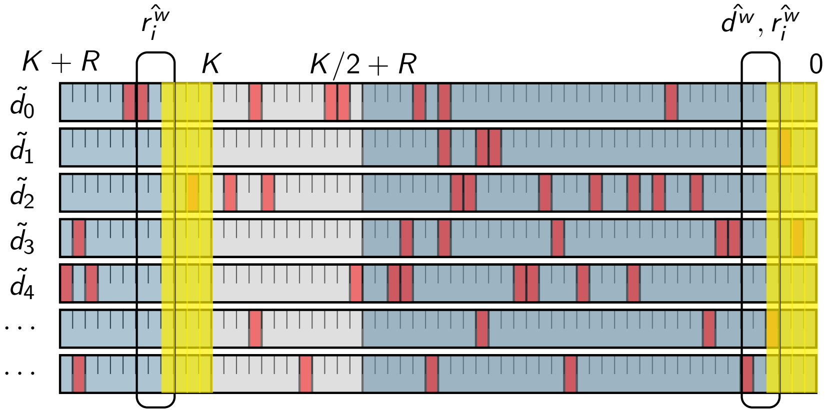

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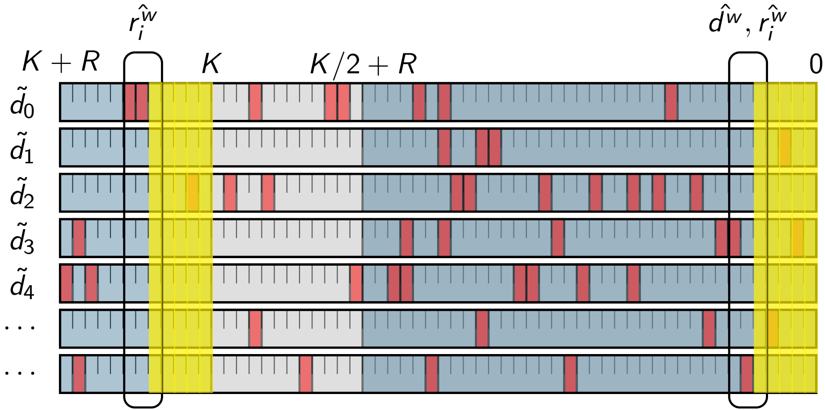

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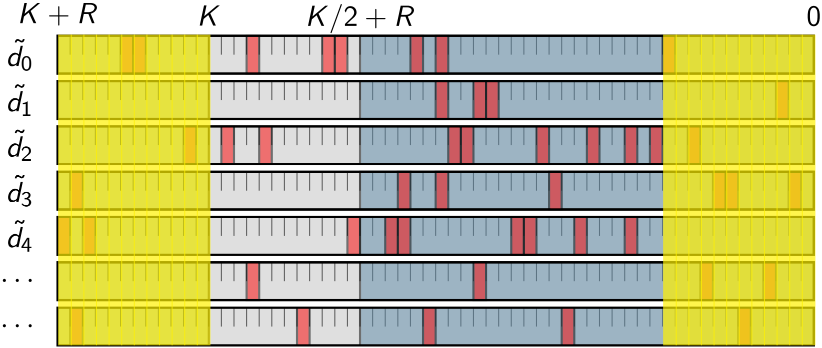

$(r_0, r_1, \cdots), d^R$

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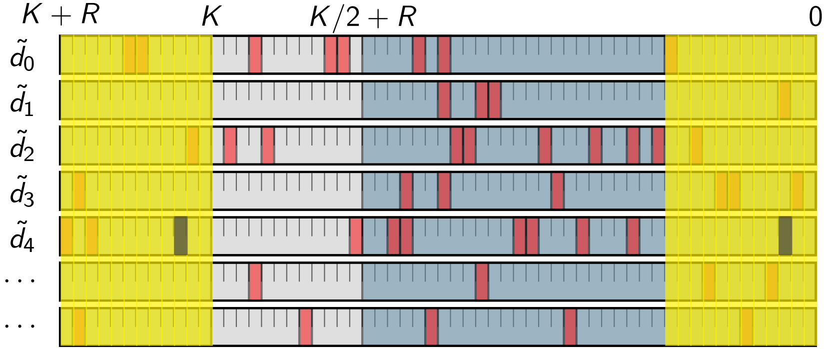

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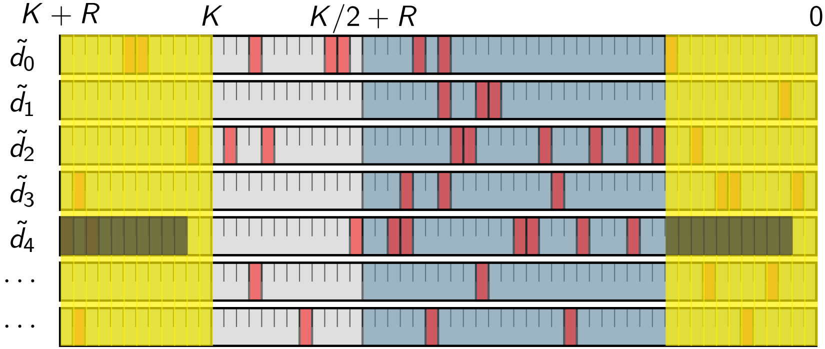

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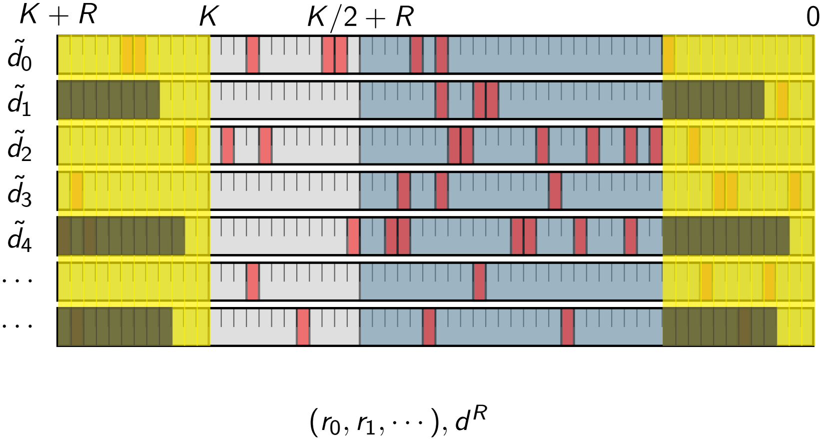

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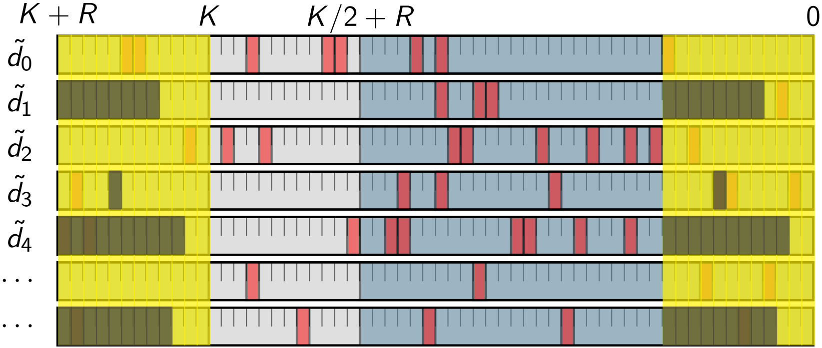

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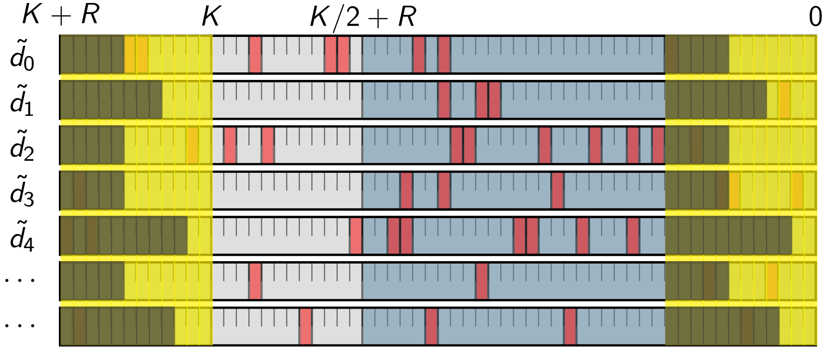

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Simulation Results [Schindler-Wiemers 2014]

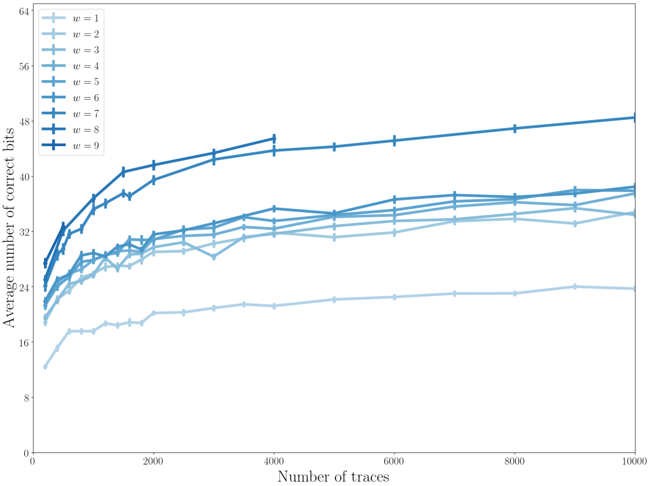
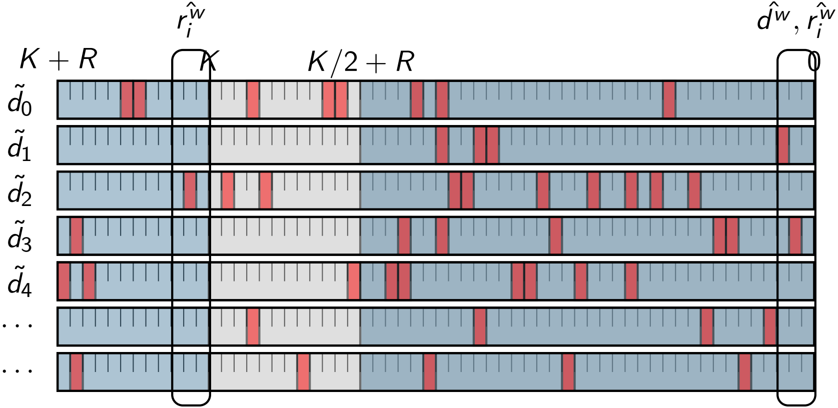


Figure: $K = 256, R = 64, \epsilon_b = 0.15$ on curve `secp-256-k1`.

Proposed Improvements on Schindler-Wiemers' Algorithm



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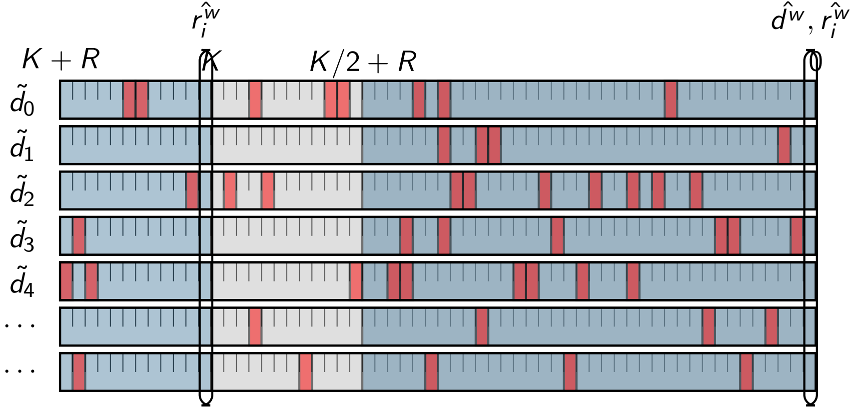

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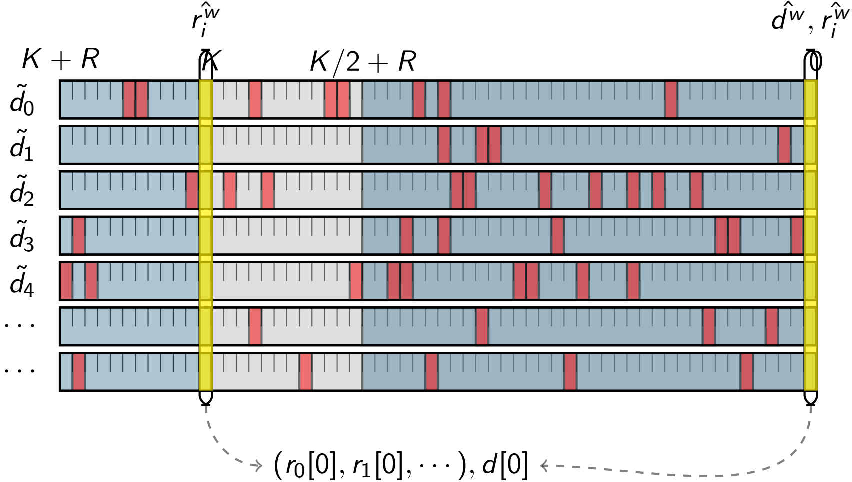

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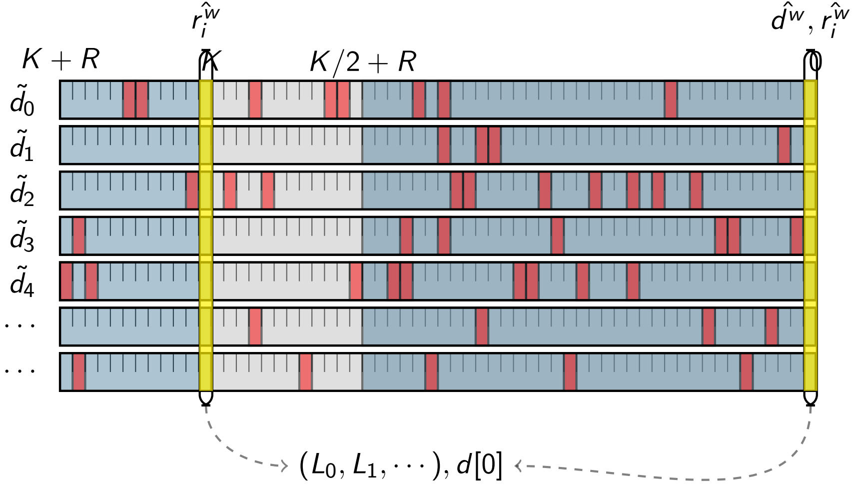

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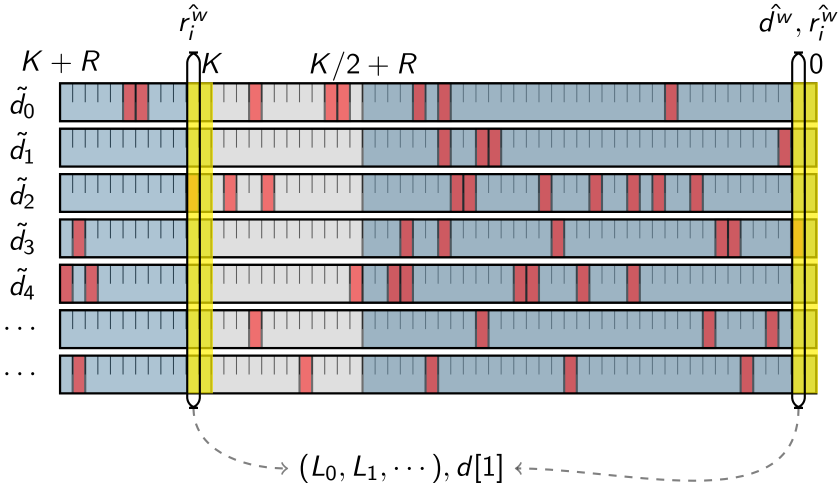

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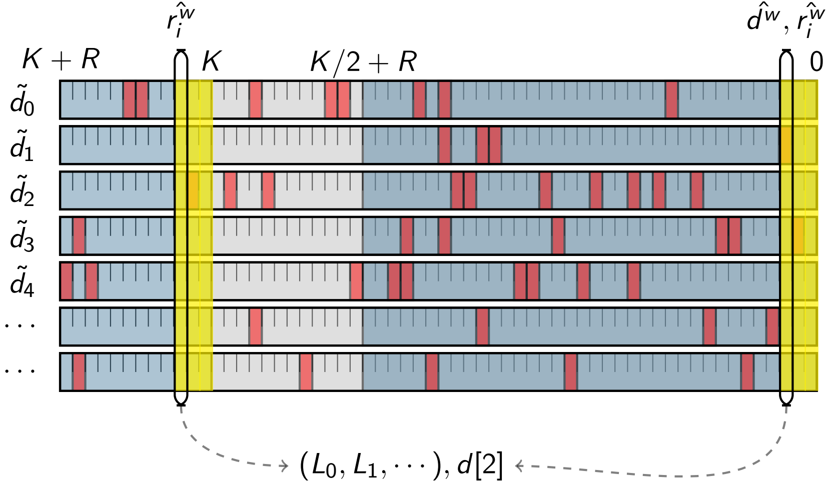

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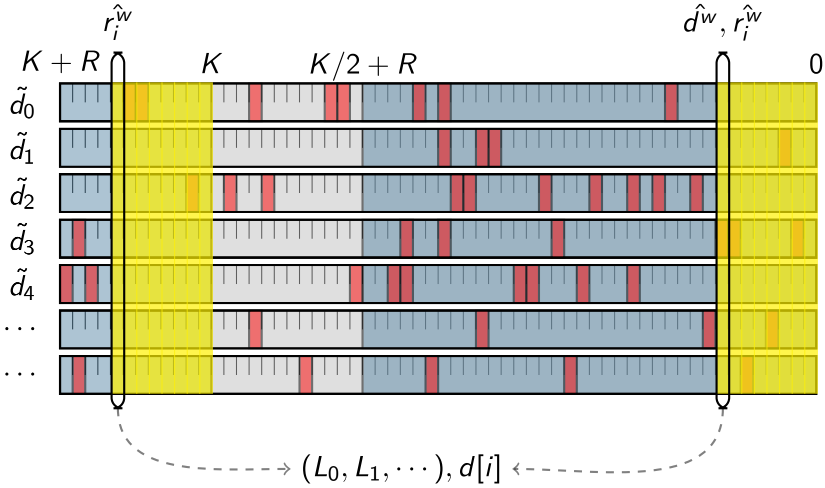
Proposed Improvements on Schindler-Wiemers' Algorithm



Complexity: $O(RNL)$ - L_ℓ stores the most probable values for r_ℓ, $\#L_\ell \leq L$

Simulation Results

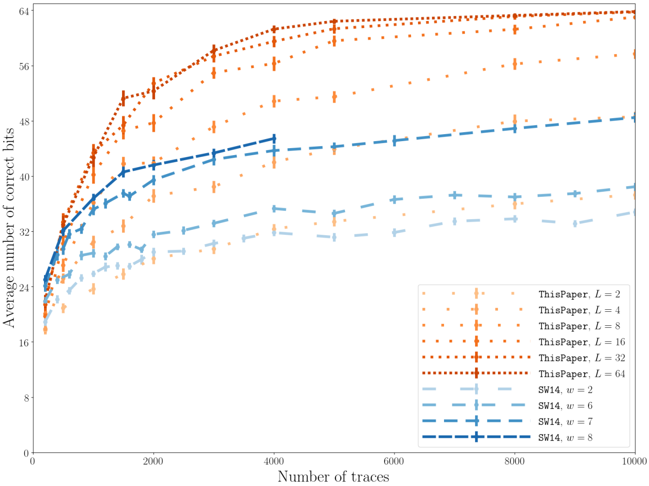


Figure: $K = 256, R = 64, \epsilon_b = 0.15$ on curve secp-256-k1.

Simulation Results

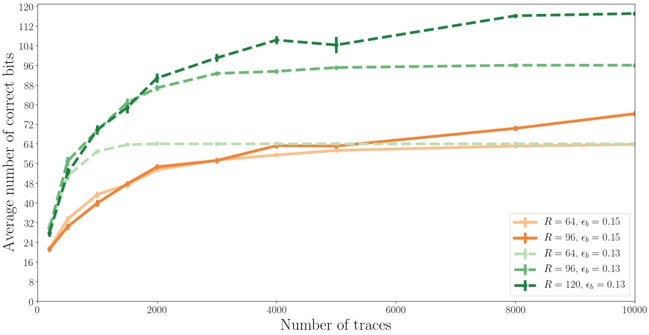
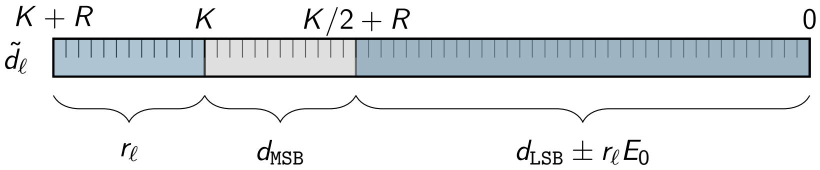


Figure: $K = 256, L = 32, t = 16.$

Structured-Order Effect on Scalar Randomization

- (d, E) of length K, r_ℓ of length R (with $R < K/2$)
- $d_\ell = d + r_\ell \times E$
- $E = 2^K \pm E_0$

$$d_\ell = r_\ell \times 2^K + d \pm r_\ell \times E_0$$



Outline

Conclusion And Future Directions

What we have seen

- The problem of blinded scalar correction is a critical problem in real-wolrd side-channel attacks.
- For structued-order Elliptic Curves taking $R < K/2$ is a clearly a bad idea.

Next Steps

- Find theoretic bounds $B(N)$ on the bit-error probability s.t. if $\epsilon_b < B(N)$ then correction is possible with N observations.
- Correct algorithm for random-order Elliptic Curves...